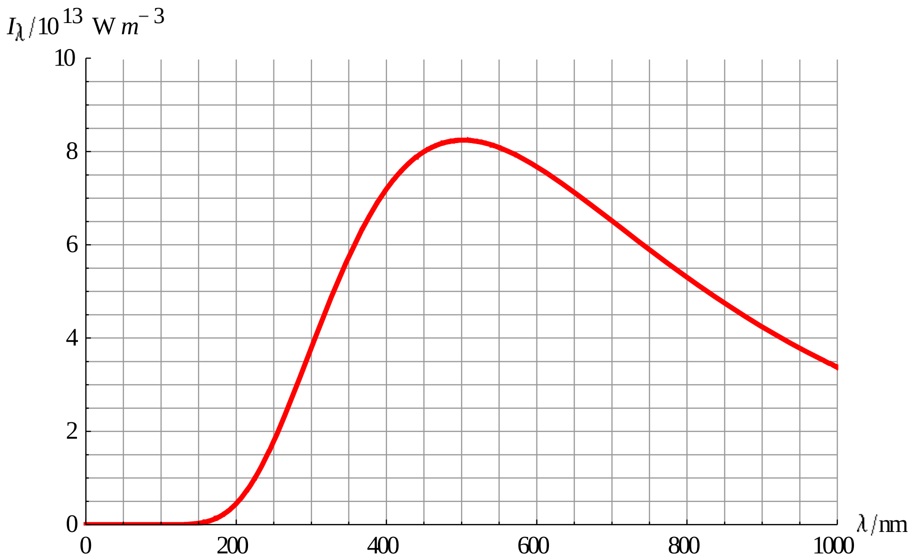
Teacher notes
Topic B

A black body spectrum worksheet

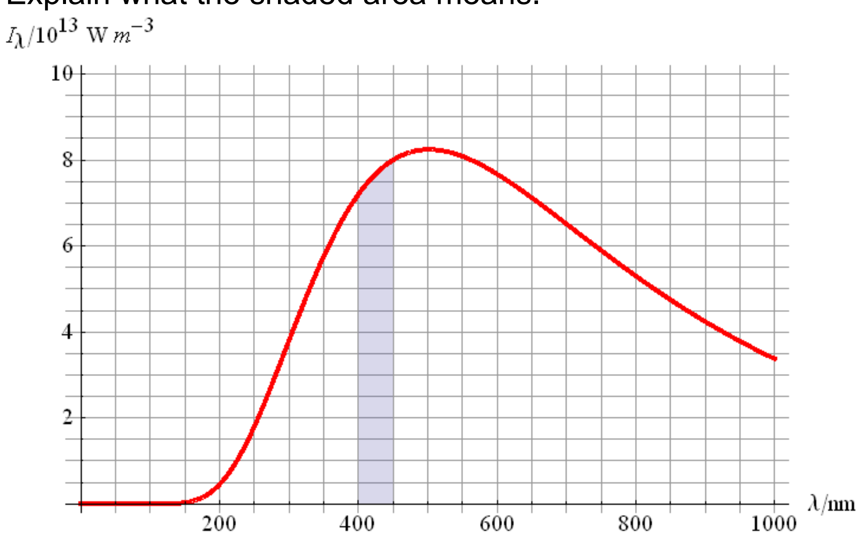
The graph shows a typical black body spectrum for a spherical black body (the Sun).



The vertical axis measures intensity per unit wavelength interval. Intensity is measured in W m^{-2} and so the unit on the vertical axis is W m^{-3}.

Use this graph to answer these questions:

(a) Determine the surface temperature of the black body.
(b) Explain what the shaded area means.



(c) Estimate the shaded area including its unit.
(d) Determine the total intensity radiated by the Sun.
(e) Estimate the intensity emitted as visible light (400 nm to 700 nm).
(f) State the ratio $\frac{\text{intensity in visible light}}{\text{total intensity}}$.
(g) The radius of the Sun is 6.96×10^{8} m. Estimate the total power radiated.
(h) If the surface area of the Sun were to increase what change would occur in the original graph?
(i) The surface temperature of the star Antares is 3500 K. The peak intensity per unit wavelength for Antares is 0.67×10^{13} W m^{-3}. Draw the black body spectrum graph for Antares on the same axes as those for the Sun.
(j) The luminosity of Antares is $6.5\times10^{4}L_{\odot}$. Determine the radius of Antares in terms of the radius of the Sun.
(k) Comment on your result in (j)

Answers

(a) The peak wavelength is read off at 500 nm so the temperature of the body is by Wien's law: $T = \frac{2.9\times10^{-3}}{500\times10^{-9}} = 5800\text{ K}$.

(b) The shaded area is the total intensity emitted in the wavelength range 400 nm to 450 nm.

(c) The shaded area is approximately a rectangle of base 50 nm and height $\frac{8.0+7.5}{2}\times10^{13} = 7.75\times10^{13}\text{ W m}^{-3}$. The area is then

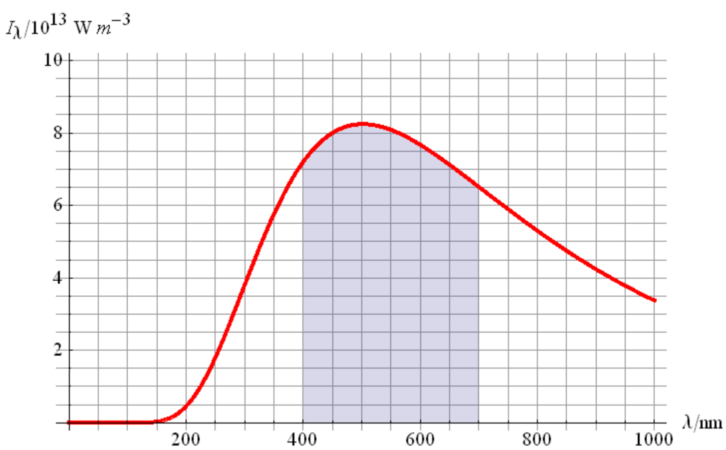
$7.75\times10^{13}\text{ W m}^{-3}\times50\times10^{-9}\text{ m} = 3.9\times10^{6}\text{ W m}^{-2}$.

This is the intensity of the emitted radiation with wavelengths in the range 400 nm to 450 nm.

(d) The entire area under the curve from zero to infinite wavelength is the total intensity emitted by the Sun. We know that the total intensity emitted is $I = \sigma T^4$ and so this must be

$I = 5.67\times10^{-8}\times5800^4 = 6.42\times10^{7}\text{ W m}^{-2}$

(e) We need to estimate this shaded area:



Counting squares we find about 93 whole squares. Each has area $0.50\times10^{13}\times50\times10^{-9} = 2.5\times10^{5}\text{ W m}^{-2}$. Hence the total area is

$93\times2.5\times10^{5} = 2.325\times10^{7}\text{ W m}^{-2}$.

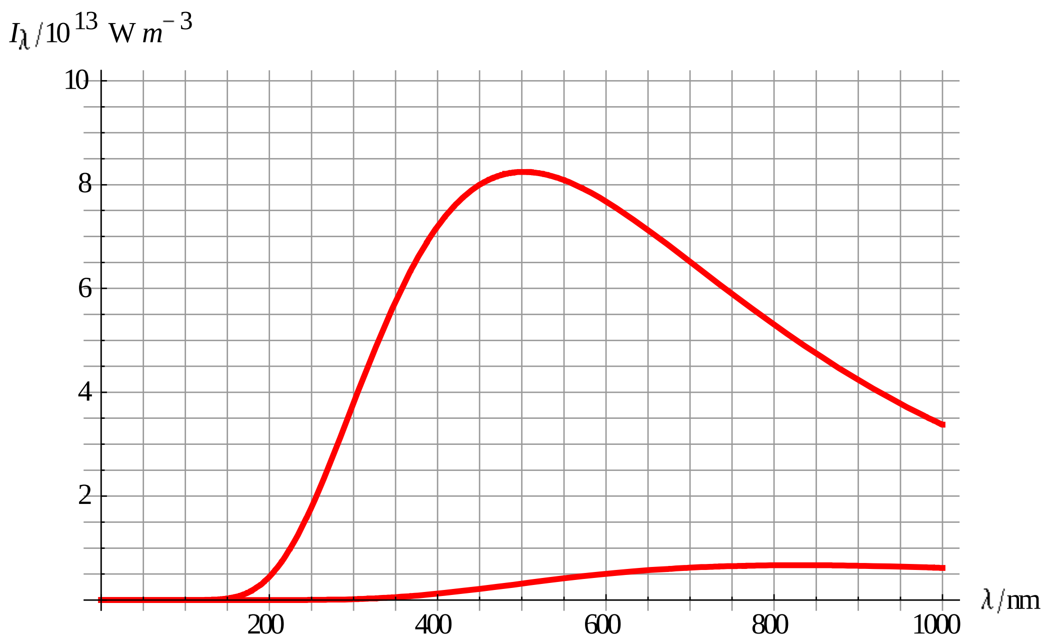
(f) Hence $\frac{\text{intensity in visible light}}{\text{total intensity}} = \frac{2.32\times10^{7}}{6.42\times10^{7}} = 0.36$.

(g) The radius is 6.96×10^{8} m and so the total power radiated (the luminosity) is

$$L = 6.42\times10^{7}\times4\pi\times(6.96\times10^{8})^{2} = 3.9\times10^{26}\text{ W}.$$

(h) No change at all. The graph is a graph of intensity (per unit wavelength) i.e. power per unit area (per unit wavelength). Increasing the area increases the power by the same amount so the ratio stays the same.

(i) The peak wavelength will be at $\lambda = \frac{2.9\times10^{-3}}{3500} = 8.3\times10^{-7}\text{m}$.



(j) $L_A = 6.5\times10^{4}L_\odot$ and so $\frac{4\pi R_A^2\sigma T_A^4}{4\pi R_\odot^2\sigma T_\odot^4} = 6.5\times10^{4}$. Hence

$$\frac{R_A}{R_\odot} = \sqrt{6.5\times10^{4}\times\left(\frac{5800}{3500}\right)^{4}} = 700.$$

(k) The peak intensity of Antares is is 0.67×10^{13} W m^{-3} which is much smaller than that of the Sun. Yet, the luminosity of Antares is much bigger than that of the Sun. This is because the surface area of Antares is enormous compared to that of the Sun.